Mini Homework 4
Name: - Math 117 - Summer 2022

1) Let $T : \mathbb{R}^3 \to M_{2\times 2}(\mathbb{R})$ be a linear map and suppose the dual map has matrix (with respect to the standard basis of both vector spaces)

$$[T^*] = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 1 & 8 & 4 & 0 \end{pmatrix}$$

(a) (2 points) Let $m_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ be the second standard basis vector in $M_{2\times 2}(\mathbb{R})$. Write $T^*(m_2^*)$ as a sum of the dual basis vectors in $(\mathbb{R}^3)^*$ (Hint: recall how matrices of linear transformations are constructed: what are the columns?)

(b) (1 point) Using part a, what is $\left(T^*(m_2^*)\right)\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$

(c) (2 points) What is the matrix of T with respect to the standard basis of both vector spaces

(d) (1 point) What is $T(\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix})$

Solution:

2) (2 points) Let V_1, V_2, W be vector spaces over $\mathbb{F}$. Show that the set $Bil(V_1 \times V_2, W)$ of bilinear maps is a vector space under point-wise addition/scalar multiplication (ie: given f, g bilinear define $f + g$ to be $(f + g)(v_1, v_2) := f(v_1, v_2) + g(v_1, v_2)$ and similarly for scalar multiplication)

Solution:

3) (2 points) Use the universal property of the tensor product to show that: given linear maps $T_1 : V_1 \to W_1$ and $T_2 : V_2 \to W_2$ we get a well defined linear map

$$T_1 \otimes T_2 : V_1 \otimes V_2 \to W_1 \otimes W_2$$

with the property that $(T_1 \otimes T_2)(v_1 \otimes v_2) = T_1(v_1) \otimes T_2(v_2)$ for all $v_1 \in V_1, v_2 \in V_2$

Solution:

4) (2 points) Let V, W be finite dimensional vector spaces, and suppose that dim(V)=dim(W). Prove that a linear transformation $T : V \to W$ is injective $\iff$ it is surjective.

Solution: